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The evolution of awareness ☆

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ABSTRACT

We consider an evolutionary approach to how awareness is determined in games where players are not necessarily aware of all possible strategies. We begin with the standard notion of evolutionarily stable equilibrium, in which potential players pursue a fixed strategy. This constitutes a minimal level of awareness, since players are not required to know anything about the game or to reason about their opponents. We then consider the introduction of players with greater awareness.

1. Introduction

Until recently, most game-theoretic representations of strategic interactions have begun with the presumption that the structure of the game, and the identity of participants, is common knowledge. However, a number of authors have recently considered problems where participants in a game may be unaware of the rules, of possible moves and of the degree to which other players may also be partially or completely unaware of the game structure (Grant & Quiggin, 2013; Halpern & Re~go, 2012; Heifetz, Meier, & Schipper, 2006). As these authors show, standard equilibrium concepts including Nash equilibrium and sequential equilibrium may be generalized to apply to games where some or all players may be partially unaware of the game structure.

In games where players are not necessarily aware of all possible strategies, it is necessary to consider how awareness is determined. In this paper, we consider an evolutionary approach to this question. We begin with the standard notion of evolutionarily stable population (ESP), in which potential players pursue a fixed strategy. This constitutes a minimal level of awareness, since players are not required to know anything about the game or to reason about their opponents.

We then consider the introduction of players with greater awareness. Increased awareness has two components, required if it is to yield non-trivial changes in the ESP. First, players must be aware of multiple strategies available to them. Second, they must have some capacity to distinguish between different kinds of opposing players. As we will prove more formally,

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unless both these conditions are satisfied, the frequency with which strategies are played will be the same, in equilibrium, with or without increased awareness.

The problem of increased awareness has previously been considered by Robson (1990), who focuses primarily on equilibrium selection issues. Robson first considers a coordination game with two Pareto-ranked pure strategy equilibria. If the initial equilibrium is inefficient, entrants who are aware of both possible equilibrium strategies, and who can recognize each other, will drive the population to the socially efficient (Pareto-dominant) equilibrium.

In this paper we consider a broader class of games, and examine some different questions:

(1) Can an initial evolutionary stable equilibrium be sustained, given the possibility of entry for more aware players?
(2) Will more aware players completely displace less aware players?
(3) Does the introduction of more aware players increase the equilibrium payoff?

We show that the answer to the first question is negative - more aware players will always be able to enter. The intuition is that, when the number of such players is small, they can always do at least as well as in the initial ESP by mimicking one of the less aware types.

The answer to the second question is positive for the case of constant sum games. In general, however, the answer is negative, as we show by example. For the third question we show by example that the equilibrium payoff may increase, decrease or remain unchanged. This result contrasts with the discussion in Robson (1990) which focuses on the 'evolution of cooperation'.

2. Background

The pursuit of a fixed strategy, with no forethought or consideration of the possible responses of others, is the epitome of bounded rationality. It is, therefore, of interest to consider the results of introducing players who are aware of different possible strategies and can vary them according to the type of the player they encounter.

In this context, awareness has two components. First, players must be able to vary their own strategy. Second, they must be able to discern, partially or completely, the type of their opponent, and respond accordingly.

The simplest way to model the second requirement is to suppose that more aware players can immediately detect the type of less aware opponents. Robson (1990) proposes a 'secret handshake', which allows more aware players to recognize each other.

An alternative approach is to suppose that any pair of players matched against each other interact repeatedly. The simplest case is that of a player who is aware of all possible strategies and who enters a population where each unaware type is confined to a single strategy. In this case, a single interaction is sufficient to determine the type of any unaware player and respond accordingly. As more aware players enter, they encounter other aware players with positive probability. As soon as an aware player varies their action they reveal their type to their opponent. If the number of interactions between a given pair of players (if positive) is large, and that there is no discounting, so that interactions taking place before the type is known may be disregarded.

3. Model

3.1. Evolutionary stable populations in the absence of awareness

As a baseline for comparison, we consider an environment in which individuals drawn at random from a large population play a two-person symmetric game with opponents drawn randomly from the same population. The set of types of the individuals is denoted by T, with generic types s, t, etc. Since the game is symmetric, the payoffs can be characterized by a $T \times T$ matrix V with typical entry $v(s,t) \in \mathbb{R}$, where $v(s,t)$ represents the payoff to a player of type s when matched with a player of type t.

Let $\Delta(T)$ denote the set of populations (strictly speaking, population distributions) of types. Formally, it is the set of functions $\mu : T \to [0,1]$, for which $\sum_{t \in T} \mu(t) = 1$. With slight abuse of notation, we shall let each $s \in T$ denote the homomorphic population in which all individuals are of type s.

For any pair of populations μ, μ' in $\Delta(T)$, set

$$EV(\mu, \mu') := \sum_{s \in S} \sum_{t \in T} v(s,t)\mu(s)\mu'(t).$$

Definition 1 (*Evolutionary stable population (ESP)*). The population $\mu \in \Delta(T)$, is evolutionarily stable if for all $\mu' \neq \mu$:

1. $EV(\mu, \mu) \geqslant EV(\mu', \mu)$ and,
2. $EV(\mu, \mu) = EV(\mu', \mu) \Rightarrow EV(\mu, \mu') > EV(\mu', \mu')$.

The standard interpretation is that the behavior of each type is genetically determined, and that higher payoffs for a given type lead to greater reproductive success therefore resulting in an increase in the proportion of that type in the population. It follows from the bilinearity of the payoff $EV(\cdot,\cdot)$, that the first condition of evolutionary stability entails all types appearing with positive probability have the same (maximal) expected payoff. The second condition rules out entry by a sub-population of types that might be as good as the equilibrium mix when matched against that mix, and better when matched against itself.

An ESP is not, in general, unique. In particular, consider any 'coordination' game in which for any pair of distinct types s and t, $v(s,t) < v(t,t)$. That is, in any mixed interaction, every type of player does worse paired with a player of another type than that type would do when paired with one of its own type.In this case, a homomorphic population of any type is evolutionarily stable.

This example raises the question of how to interpret the equilibrium payoff in an ESP. Consider, for example, a coordination game with two types s, t such that $v(t,s) = v(s,t) < v(s,s) < v(t,t)$. Homomorphic populations of either type s or t are evolutionarily stable, but the payoff is higher for a population of type t. In the usual economic interpretation with the outcome interpreted as a Nash equilibrium of a normal-form game, the s equilibrium is a Pareto-dominated 'trap'.

The interpretation in terms of evolutionary biology is far less clear. One possible interpretation of the payoff is the population that can be sustained in a given environment. But this is beyond the scope of the current paper.

3.2. Introducing differential awareness

We now extend the model to allow for players who are aware of more than one strategy. Given a set of 'pure' types T, awareness may be modelled by the construction of a set $\mathcal{T} \subseteq 2^T - \emptyset$, with typical elements $\sigma, \tau \subseteq T$. That is, each type in a game with awareness has access to a non-empty subset of the strategies available to pure types. We then construct the matrix V with typical element $\mathfrak{v}(\sigma,\tau)$, subject to the requirement that $\mathfrak{v}(\sigma,\tau)$ represents the Nash equilibrium payoff of the row player from a game where the row player has the strategy set σ, the column player has the strategy set τ and the payoffs are derived from the relevant submatrix of V. We will require that, for the case $\sigma = \tau$, the equilibrium must be symmetric. That is,

$$\begin{aligned} \mathfrak{v}(\sigma,\tau) = EV(\mu,\mu'),\ &\text{for some } \mu \in \Delta(\sigma) \text{ and } \mu' \in \Delta(\tau) \text{ (and with } \mu = \mu' \text{ if } \sigma = \tau) \\ &\text{such that } EV(\mu,\mu') \geqslant EV(\hat{\mu},\mu'),\ \text{for all } \hat{\mu} \in \Delta(\sigma), \\ &\quad \text{and } EV(\mu',\mu) \geqslant EV(\hat{\mu}',\mu),\ \text{for all } \hat{\mu}' \in \Delta(\tau). \end{aligned}$$

Notice that unless all the games derived in this way have a unique Nash equilibrium, the payoff matrix $\mathcal{V}$ is not uniquely determined by the original payoff matrix V.

A game defined in this way will be described as an *evolutionary game with differential awareness*. Observe that the awareness of the players may be characterized by the partial ordering of set inclusion. If we adjoin the empty set $\emptyset$ to $\mathcal{T}$, this partial ordering generates a lattice with the union and intersection as the join and meet operations.

If we disregard the structure of $\mathcal{T}$, an evolutionary game with differential awareness may be regarded simply as a normal-form game to which the standard definitions of Nash equilibrium and evolutionarily stable population apply.

3.3. Characterization: the case of zero-sum games

In the particular case where the matrix V characterizes a zero-sum symmetric game, that is, where $v(s,t) + v(t,s) = 0$ for all $(s,t) \in T \times T$, the payoff matrix V is well-defined since for any pair σ, τ in T, we have

$$\mathfrak{v}(\sigma,\tau) = \max_{\mu\in\Delta(\sigma)} \min_{\mu'\in\Delta(\tau)} EV(\mu,\mu').$$

That is, $\mathfrak{v}(\sigma,\tau)$ is the unique value for the row player of the game in which the row player has the strategy set σ, the column player has the strategy set τ and the payoffs are derived from the relevant submatrix of V. Since the game is symmetric it also follows that $\mathfrak{v}(\sigma,\sigma) = 0$, for all σ in T.

Let $\mu^* \in \operatorname{argmax}_{\mu\in\Delta(\sigma)} \left(\min_{\mu'\in\Delta(\tau)} EV(\mu,\mu')\right)$ denote a maximinimizer (or prudent) strategy and set supp $\mu^* := \{t \in T : \mu^*(t) > 0\}$ to be its support. Following Kaplansky (1945) we say the game V is completely mixed if supp $\mu^* = T$. Kaplansky's Theorem 5 provides necessary and sufficient conditions on V for the game to be completely mixed. One implication of those conditions is that a zero-sum symmetric game cannot be completely mixed if $|T|$ is odd.[1] Conversely, Duersch, Oechssler, and Schipper (2012, Observation 4, p. 556) show that a symmetric two-player zero-sum game has a pure equilibrium if and only if it is not what they refer to as a generalized Rock, Paper, Scissors (RPS) game. A generalized RPS game is one in which for each column of V there exists a row with a strictly positive payoff.

[1] One of these conditions is that the rank of V must be $|T| - 1$. But since the rank of a skew-symmetric sqaure matrix is even this means that any 2×2 zero-sum symmetric game must have a pure equilibrium.

As the next proposition demonstrates, in an evolutionary game with differential awareness, evolutionary pressures push toward players who are sufficiently aware to play a maximinimizer strategy.

Proposition 2. *For a generalized RSP zero-sum game, where the potential population consists of fully aware players and single-strategy types, the unique ESP of the associated evolutionary game with differential awareness consists entirely of fully aware players.*

Proof. For all $t \in T, \mathfrak{v}(T, \{t\}) = \max_{s \in T} v(s,t) > 0$, since V is a generalized RPS game. Thus for no $t \in T$, is $(\{t\}, \{t\})$ a Nash equilibrium of the evolutionary game with differential awareness. Furthermore, by skew-symmetry $\mathfrak{v}(\{t\}, T) = -\,\mathfrak{v}(T, \{t\}) < 0$ $(= \mathfrak{v}(T,T))$, for all $t \in T$. Thus (T,T) is a strict Nash equilibrium of the evolutionary game with differential awareness which makes it ES. □

3.4. Characterization: the general case

Intuitively, it seems evident that greater awareness must be advantageous, and lesser awareness disadvantageous. The analysis of zero-sum games is consistent with this intuition. The general case is more complex. Consistent with intuition, no ESP with only unaware players can exist in a game with differential awareness. Trivially, the entry of fully aware types must reduce the population proportion of unaware types in aggregate.

Proposition 3. *Any ES polymorphic population of a game with unaware players cannot be an ES population of the associated evolutionary game with differential awareness in which the potential population contains fully aware types.*

Proof. Let μ^* denote the ES of the game V. From the definition of ES, it follows $EV(\mu^*, \mu^*) = EV(t, \mu^*) = EV(\mu^*, t) > EV(t,t)$ for all $t \in \text{supp } \mu^*$. So in the associated evolutionary game with differential awareness, for the fully aware type T we have $\mathfrak{v}(T, \{t\}) = \max_{s \in T} v(s,t) > EV(\mu^*, t)$ for all $t \in \text{supp } \mu^*$. □

On the other hand, as we will show, some unaware types may not only survive but prosper, in the sense that their population share increases in a game with differential awareness relative to the associated game with only unaware players.

Proposition 4. *There exist ESP of games with differential awareness in which unaware players survive with positive probabilities. Further, some unaware types may increase their share of the population relative to the ESP of the game with only unaware players*

Proof. See Example 1 below. □

A second hypothesis, suggested by the analysis of Robson (1990) is that the introduction of fully aware players will improve the equilibrium payoff, in particular, by encouraging the selection of Pareto-dominant equilibria. This beneficial outcome occurs in some cases. However, it is also possible that increased awareness may reduce the equilibrium payoff.

Proposition 5. *The introduction of more aware players may increase or reduce the equilibrium payoff.*

Proof. See Examples 2 and 3. □

One way of interpreting the results derived here is that unawareness may be seen as equivalent to credible commitment. A player who is only aware of one strategy cannot deviate from that strategy. As is well known, credible commitment of this kind can be beneficial to the players concerned.

4. Examples

4.1. 1. Coordination games

We first consider a simple coordination game, with a Pareto-dominant equilibrium.

Payoffs to Row

	a	b
a	1	0
b	0	2

Both (a,a) and (b,b) are strict Nash equilibria, and so both a and b are monomorphic ES populations. They are also generalized ESP equilibria in the case $\mathcal{T} = \{\{a,b\}\}$, that is, when $\mathcal{T}$ contains only the fully aware type with access to both strategies a and b.

However, in the case $\mathcal{T} = \{\{a\},\{b\},\{a,b\}\}$ there are three evolutionary games with differential awareness associated with the coordination game, one for each of its three symmetric NE, $(a,a), (b,b)$ and $((2/3,1/3),(2/3,1/3))$, respectively.

I: $\{a,b\}$ plays a against itself

$\mathfrak{v}(\sigma,\tau)$	$\{a\}$	$\{b\}$	$\{a,b\}$
$\{a\}$	1	0	1
$\{b\}$	0	2	2
$\{a,b\}$	1	2	2

II: $\{a,b\}$ plays b against itself

$\mathfrak{v}(\sigma,\tau)$	$\{a\}$	$\{b\}$	$\{a,b\}$
$\{a\}$	1	0	1
$\{b\}$	0	2	2
$\{a,b\}$	1	2	1

III: $\{a,b\}$ plays $(2/3, 1/3)$ against itself

$\mathfrak{v}(\sigma,\tau)$	$\{a\}$	$\{b\}$	$\{a,b\}$
$\{a\}$	1	0	1
$\{b\}$	0	2	2
$\{a,b\}$	1	2	2/3

In I an ESP does not formally exist, although any population of fully aware types and unaware type $\{b\}$ are not vulnerable to invasion by any other potential sub-population. In II and III, however, the unique ES is a monomorphic population of the unaware type $\{b\}$.

As this example illustrates, the (potential) existence of the fully aware type can exclude the Pareto-dominated ESP that can arise with either group alone.

4.2. 2. Hawk-Dove Game

Consider a symmetric Hawk-Dove game, with the payoffs to the Row player described in the table below.

Payoffs to Row

	H	D
H	$\frac{v-c}{2}$	v
D	0	$\frac{v}{2}$

, $c > v > 0$.

We interpret a type H (respectively, D) player as one who is only aware of the action H (respectively, D). It is straightforward to check the only ESP is a polymorphic population $\mu^0 = (v/c, 1 - v/c)$. The expected payoff from playing Hor Dagainst an opponent randomly drawn from a population with proportions μ^0 is $[(c - v)v]/[2c]$.

Now consider the introduction of a "sophisticated" player $\{H,D\}$ who is aware of both actions H and D, and can identify other types of players. Against an $\{H\}$ (respectively, $\{D\}$) player, $\{H,D\}$ will play D(respectively, H). If matched against another sophisticated player, we assume they randomize $(v/c, (c - v)/c)$ over their two actions, yielding an expected payoff of $[(c - v)v]/[2c]$. The payoffs of this associated evolutionary game with differential awareness are thus:

Payoff to Row

$\mathfrak{v}(\sigma,\tau)$	$\{H\}$	$\{D\}$	$\{H,D\}$
$\{H\}$	$\frac{v-c}{2}$	v	v
$\{D\}$	0	$\frac{v}{2}$	0
$\{H,D\}$	0	v	$\frac{(c-v)v}{2c}$

Notice $\{H,D\}$weakly dominates $\{D\}$, so D will not appear in the ESP. The unique symmetric mixed strategy equilibrium is (μ^*, μ^*) where:

$$\mu^* = \left(\frac{v(c+v)}{c^2+v^2}, 0, \frac{c(c-v)}{c^2+v^2}\right).$$

The expected payoffs to the three types against the polymorphic population μ^* are:

$$\mathfrak{v}(\{H\},\mu^*) = \mathfrak{v}(\{H,D\},\mu^*) = \frac{v(c-v)^2}{2(c^2+v^2)} > 0 = \mathfrak{v}(\{D\},\mu^*).$$

Proposition 6. *The polymorphic population μ^* is the unique ESP for the evolutionary game of differential awareness associated with the Hawk-Dove game.*

Proof. First notice that for any $\mu' = (p, 0, 1 - p)$, we have

$$
\begin{aligned}
&\mathfrak{v}(\mu^*,\mu') - \mathfrak{v}(\mu',\mu') \\
&= \left(\frac{v(c+v)}{c^2+v^2} - p\right)\mathfrak{v}(\{H\},\mu') + \left(\frac{c(c-v)}{c^2+v^2} - 1 + p\right)\mathfrak{v}(\{H,D\},\mu') \\
&= \left(\frac{v(c+v)}{c^2+v^2} - p\right)\left(p\left(\frac{v-c}{2}\right) + (1-p)v\right) \\
&\quad + \left(\frac{c(c-v)}{c^2+v^2} - 1 + p\right)\left((1-p)\frac{(c-v)v}{2c}\right) \\
&= \frac{\left(c^2p + pv^2 - cv - v^2\right)^2}{2c(c^2+v^2)} > 0, \text{for all} p \neq \frac{v(c+v)}{c^2+v^2}.
\end{aligned}
$$

□

Proof. Now consider a general polymorphic population $\mu'' = (1-\alpha)\mu' + \alpha(0,1,0)$, where $\mu' = (p,0,1-p)$ and $\mu'' \neq \mu^*$, that is, either $\alpha > 0$, or $p \neq v(c+v)/(c^2+v^2)$. We have

$$
\begin{aligned}
&\mathfrak{v}(\mu^*,\mu'') - \mathfrak{v}(\mu'',\mu'') \\
&= \mathfrak{v}(\mu^*,(1-\alpha)\mu' + (0,\alpha,0)) - \mathfrak{v}(\mu'',(1-\alpha)\mu' + (0,\alpha,0)) \\
&= (1-\alpha)[u(\mu^*,\mu') - u(\mu'',\mu')] + \alpha[\mathfrak{v}(\mu^*,\{D\}) - \mathfrak{v}(\mu'',\{D\})]
\end{aligned}
$$

Now since $\mathfrak{v}(\mu^*,\{D\}) = v$ and $\mathfrak{v}(\mu'',\{D\}) \leqslant v$ (with strict inequality if $\alpha > 0$), it follows $\mathfrak{v}(\mu^*,\{D\}) - \mathfrak{v}(\mu'',\{D\}) \geqslant 0$ (and with *strict* inequality if $\alpha > 0$.) And for the first term

$$
\begin{aligned}
&\mathfrak{v}(\mu^*,\mu') - \mathfrak{v}(\mu'',\mu') \\
&= (1-\alpha)[\mathfrak{v}(\mu^*,\mu') - \mathfrak{v}(\mu',\mu')] + \alpha[\mathfrak{v}(\mu^*,\mu') - \mathfrak{v}(\{D\},\mu')]
\end{aligned}
$$

Since $\mathfrak{v}(\{D\},\mu') = 0$ (recall $\mu'(\{D\}) = 0$) and $\mathfrak{v}(\mu^*,\mu') \geqslant 0$, it follows from the first step in the proof that $\mathfrak{v}(\mu^*,\mu') - \mathfrak{v}(\mu'',\mu') \geqslant 0$ (and with strict inequality if $\alpha < 1$ and $p \neq v(c+v)/(c^2+v^2)$. Putting this all together, we have shown $u(\mu^*,\mu'') - u(\mu'',\mu'') > 0$, whenever either $\alpha > 0$ or $p \neq v(c+v)/(c^2+v^2)$, that is, whenever $\mu'' \neq \mu^*$. □

Comparing the original ES polymorphic population to the new one, we have

$$
\mu^*(\{H\}) - \mu^0(H) = \frac{v(c+v)}{c^2+v^2} - \frac{v}{c} = \frac{v(c^2-v^2)}{c(c^2+v^2)} > 0.
$$

That is, the introduction of the more aware type of player, drives out the Doves and leads to a higher evolutionary stable proportion of Hawks and thus a *lower* expected payoff for all individuals in the population!

4.3. 3. Australian Battle-of-the-Sexes[2]

Payoffs to Row

	a	b
a	0	2
b	1	0

Again we interpret a type a(respectively, b) player as one who is only aware of the action a(respectively, b). It is straightforward to check the only ESP is a polymorphic population $\nu^0 = (2/3, 1/3)$. The expected payoff from playing $a$or $b$against an opponent randomly drawn from a population with proportions ν^0 is thus 2/3.

As we did above in the Hawk-Dove game, let us now consider the introduction of a "sophisticated" player s who is aware of both actions a and b, and can identify other types of players. Against a type a (respectively, type b) player, s will play b (respectively, a). If matched against another sophisticated player, for now assume they randomize two-thirds/one-third over the two actions $a$and b. That is, they play the unique symmetric mixed strategy of the basic stage game, yielding an expected payoff of 2/3. The payoffs of this augmented game are thus:

Payoff to Row

$\mathfrak{v}(\sigma,\tau)$	a	b	$\{a,b\}$
a	0	2	2
b	1	0	1
$\{a,b\}$	1	2	2/3

[2] So named by Binmore (1992) since the payoffs are all "upside down" compared to the original battle of the sexes game.

Notice that s does not weakly dominate b. The unique mixed symmetric mixed strategy is $v^* = (8/15, 1/15, 6/15)$. The expected payoffs to the three types against such a polymorphic population are $u(a, v^*) = u(b, v^*) = u(s, v^*) = 14/15$. It is also the ESP, since

$$\begin{aligned} u(v^*, a) &= 7/15 > 0 = u(a, a), \\ u(v^*, b) &= 28/15 > 0 = u(b, b), \text{ and} \\ u(v^*, s) &= 29/15 > 2/3 = u(s, s). \end{aligned}$$

In summary, the introduction of the more aware type of player, drives down the myopic-types $a$and b, replacing them with sophisticated types who get a higher payoff when they meet themselves, thus in the new ESP all individuals in the population enjoy a higher expected payoff than before.

As a slight alternative, if we assume the sophisticated types can secretly signal their type to each other and by way of those signals coordinate on the off-diagonal payoffs, so enjoying an expected payoff of 3/2, then the payoffs of the augmented game become

Payoff to Row

$\mathfrak{v}(\sigma, \tau)$	$\{a\}$	$\{b\}$	$\{a,b\}$
$\{a\}$	0	2	2
$\{b\}$	1	0	1
$\{a,b\}$	1	2	**3/2**

Now $\{a,b\}$ weakly dominates $\{b\}$, and the unique symmetric mixed strategy equilibrium is $\hat{v} = (1/3, 0, 2/3)$. The expected payoffs to the three types against such a polymorphic population are $u(a, \hat{v}) = u(s, \hat{v}) = 4/3 > 1 = u(b, \hat{v})$. Now since for any $\mu' = (p, 0, 1-p)$, we have $\mathfrak{v}(\mu', \hat{v}) = 4/3 = \mathfrak{v}(\hat{v}, \hat{v})$, to establish $\hat{v}$ is ESP we need to show $\mathfrak{v}(\hat{v}, \mu') > \mathfrak{v}(\mu', \mu')$.

But

$$\begin{aligned} &\mathfrak{v}(\hat{v}, \mu') - \mathfrak{v}(\mu', \mu') \\ &= \left(\frac{1}{3} - p\right)\mathfrak{v}(a, \mu') + \left(\frac{2}{3} - 1 + p\right)\mathfrak{v}(s, \mu') \\ &= \left(\frac{1}{3} - p\right)2(1-p) + \left(\frac{2}{3} - 1 + p\right)\left(p + \frac{3}{2}(1-p)\right) \\ &= \frac{(3p-1)^2}{6} > 0, \text{for all} p \neq \frac{1}{3}. \end{aligned}$$

In this scenario, the introduction of more aware players has driven out the $\{b\}$types completely and reduced the prevalence of $\{a\}$ types from 1/2 to 1/3 of the population. The higher payoff that a pair of sophisticated types generate leads to an overall doubling of the expected payoff enjoyed by all members of the population.

5. Concluding comments

In this paper, we have considered differential awareness as a problem of evolutionary selection. Our primary conclusion of this paper is that greater awareness is not always better, either in the sense of individual selective fitness, or in the sense that populations with higher levels of awareness will receive higher equilibrium payoffs and therefore perform better in problems involving group selection. On the contrary, players with higher awareness may fail to drive lower awareness players out of the population, and their entry may be reduce the overall welfare of the population. On the other hand, if the population is initially unaware of some strategies, the resulting equilibrium is never stable if entry by more aware agents is possible.

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