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Note

A matter of interpretation: Ambiguous contracts and liquidated damages ☆

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ABSTRACT

We consider the optimality of liquidated damages contracts in a setting of contractual ambiguity and potential for disputes. We show that when parties are ambiguity averse enough, they will optimally choose liquidated damages contracts and sacrifice risk sharing opportunities.

1. Introduction

Language is a matter of interpretation, and interpretations will differ. This fact is of fundamental importance in the construction of contracts, which are written or verbal agreements that the parties should act in particular ways under particular conditions. For any contract to be successfully implemented, the parties must agree on whether the relevant conditions apply. If parties may differ in their interpretation of the conditions that apply including the actions that are required, contracts will lead to disputes and, ultimately, litigation.

To avoid disputes, parties to a contract may seek to avoid terms likely to give rise to dispute, even when the resulting contract is incomplete, in the sense that opportunities for risk-sharing or productive cooperation are foregone. They might also include default clauses like liquidated damages in order to avoid further litigation fees incurred during *ex post* arguments over contractual terms.

In ordinary usage, we use the term 'ambiguous' to describe language that is open to multiple interpretations. The converse term 'unambiguous' is used when language is clear, and its meaning is agreed by all. Linguistic ambiguity may be either 'syntactic', when the same sequence of words may be interpreted in different ways, or 'semantic', when individual words have more than one meaning.

In formal decision theory, the term 'ambiguity' has a distinct technical meaning, applicable in the case when individual decision makers are unable to attach a unique probability distribution over the state space. We will refer to this as

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'probabilistic ambiguity'. In this paper our focus is on linguistic ambiguity. However, the preferences we employ to analyze the effects of linguistic ambiguity closely resemble those appearing in the probabilistic ambiguity literature (in particular, Gul and Pesendorfer, 2014).

We think of the ambiguity that parties to a contract face as arising from the potentially different interpretations of the contingencies specified in the contract. Grant et al. (2012a) examined the implications of contractual ambiguity in bargaining problems, and showed that ambiguity may lead to incomplete risk sharing.

In the present paper, we consider the contractual specification of damages that apply when one party is unable (or finds it undesirable) to fulfill their contractual obligations. In this context, we consider 'liquidated damages' contracts which specify a constant payment for the case of default. We show in Proposition 1 that liquidated damages contracts are *ex ante* efficient when the aversion to ambiguity is sufficiently high. It is natural to ask, however, whether the efficiency of liquidated damages contracts obtains in the standard state-space approach. We show in Proposition 2 that it does not.

Efficiency arguments for liquidated damages clauses appear in the economics literature as far back as Shavell (1980), and are elaborated in Che and Chung (1999). The efficiency is obtained by considering effects on the *ex ante* investment and default incentives of the parties. These papers model risk neutral parties in the absence of ambiguity, so there is no contractual rationale for risk sharing or ambiguity aversion. Chung (1991) pointed out the difficulty of simple contracts being efficient when both parties are risk averse.

In contrast we consider risk averse and ambiguity averse parties. Nevertheless, we find that liquidated damages contracts can be efficient when coupled with ambiguity aversion. The source for the efficiency of liquidated damages is their security against *ex post* disputes.

Our result is based on an intuition similar to that of Mukerji and Tallon (2001) who demonstrated how probabilistic ambiguity about the idiosyncratic risk associated with financial assets may deter agents from trading such assets. In their model, however, disagreement between ambiguity averse parties arises after the signing of a contract, but before the realization of the state of nature, as each party evaluates their position according to the least favorable of a set of priors. There, ambiguity is expressed in terms of multiple priors. In our model, disagreement arises after the realization of the state of nature, when the parties are in dispute over what action is required by the contract. Here, ambiguity is expressed in terms of multiple interpretations. Given the way we formalize the linguistic ambiguity in our model, however, *mathematically* the mechanisms in both papers operate quite similarly with ambiguity inhibiting each party's willingness to contract or to adopt a contract with incomplete risk sharing.

Motivations for liquidated damages that are more in line with our approach appear in the legal literature. As argued by Hillman (2000, p. 732):

> Because people do not like ambiguity, contracting parties may prefer the safety of a liquidated damages provision over the uncertainty of expectancy damages.

Similarly, Goetz and Scott (1977, p. 557) explain:

> The expected cost of establishing true losses under conventional damage measures will thus induce parties who face uncertain or unprovable anticipated losses to negotiate stipulated damage agreements.

The efficiency of liquidated damages contracts in our model rests on the aversion to ambiguity being sufficiently pronounced to induce the parties to forgo risk sharing opportunities in default states. In general, however, efficient contracts exploit risk sharing opportunities in non-default states.

The paper is organized as follows. In Section 2 we present a formal model of contracting in the presence of linguistic ambiguity. Following Grant et al. (2012a), we adopt a convenient subclass of Gul and Pesendorfer's (2014) *expected uncertain utility theory* to model the way the parties evaluate such contracts. We then illustrate, with reference to the idea of liquidated damages, how sufficient aversion to linguistic ambiguity can lead the parties to prefer incomplete risk sharing to ambiguous contracts. We conclude with a brief discussion about how the representation of ambiguity developed in this paper suggests new approaches to further applications in contract theory and agency theory.

2. Contracts, linguistic ambiguity and preferences

Recall from the introduction, a contract is a written or verbal agreement entered into by the parties to the contract which specifies the actions to be taken under various conditions. Formally, we follow the approach developed in Grant et al. (2012a) by modeling this as a finite state space S describing all possible conditions that can be articulated in the common language used by the two parties, and an action space A that describes all actions that the parties can agree to take. Thus we identify a contract c by a mapping from S to A.[1] Let C denote the set of contracts.

While the language and hence corresponding state space S are common, the parties to a contract may disagree ex post over which contractual terms have been satisfied and thus which actions are required by the contract.

[1] Linguistic ambiguity is developed more fully in Grant et al. (2012b) where a contract is explicitly specified in a (formal) language. From the formal language we derive the state space S and, for each contract, the associated mapping from S to A.

Suppose that one party to a contract evaluates the conditions specified in the contract and concludes that the state is s. Individuals can recognize that certain conditions are ambiguous, and may be interpreted differently by the other party, so that, in the given circumstances, the other party might conclude that the actual state is s' $(\neq s)$. We model this by specifying, for each state s a 'dispute set' $D(s) \subset S$ that is assumed to contain s. The interpretation of $D(s)$ is that when a player believes s has obtained, she thinks the other player could have assessed any state s' in $D(s)$ as having obtained.

Clearly, if $D(s) = \{s\}$, the players will agree on the occurrence of state s, and the requirement for the action $c(s)$. Thus, the language available to the players unambiguously specifies state s. More generally, we say an event $E \subset S$ is *unambiguous* if $E = \bigcup_{s \in E} D(s)$. Grant et al. (2012a, Lemma 1) showed that the set of unambiguous events $\mathcal{E}_U$ is an algebra of subsets of S, that is, $\mathcal{E}_U$ contains S, and is closed under taking complements and finite intersections.

Notice that if a contract specifies the same action for every element of an unambiguous event E, then the fact that the players disagree about which element of E has occurred does not matter. Thus we say a contract c is *unambiguous* if it is measurable with respect to $\mathcal{E}_U$. For ambiguous contracts, however, when party i believes that the state is s, she considers it possible that the other party believes any element of $D(s)$ has obtained. The ambiguity here is all expressed in terms of dispute sets and due to multiple interpretations.

Turning to preferences, we first posit an outcome function $y_i : A \times S \to \mathbb{R}$, for each $i = 1, 2$, where we interpret $y_i(a, s)$ as the 'wealth equivalent' outcome for party i when action a from A is undertaken and she observes s in S.

We assume that individuals anticipate that a dispute will lead to a 'war-of-attrition' game in which each party's equilibrium payoff is equal to her security payoff, in this case, the outcome associated with the other player's interpretation.[2] That is, if party i sees state s when the other party sees s' then unless the other party *prefers* party i's interpretation to his own, the (certainty-equivalent) outcome in the war-of-attrition equilibrium for party i is $y_i(c(s'), s)$. Hence in terms of a given contract c, this possibility of dispute generates ambiguity about the action that will actually be implemented. Depending upon which interpretation is followed, the outcome might conceivably be any member of the set $\{y_i(c(s'), s)\colon\ s' \in D(s)\}$.

Following the approach developed in Grant et al. (2012a), we adopt a special case of Gul and Pesendorfer's (2014) class of *expected uncertain utility maximizers* to model the way the parties evaluate contracts subject to linguistic ambiguity that may result in potential loss arising from a dispute and the resulting war of attrition. This leads party i to evaluate a contract c according to the function:

$$V_i(c) = \sum_{s \in S} \pi_s \Big[\alpha_i \min_{s' \in D(s)} v_i\big(y_i\big(c\big(s'\big), s\big)\big) + (1 - \alpha_i) \max_{s' \in D(s)} v_i\big(y_i\big(c\big(s'\big), s\big)\big) \Big], \tag{1}$$

where $\pi \in \mathbb{R}_+^{|S|}$ satisfying $\sum_{s \in S} \pi_s = 1$, is the (common) prior probability over S, $v_i(\cdot)$ is her preference scaling function that encodes her attitudes toward risk, and $\alpha_i \in [0, 1]$ is interpreted as a measure of her (relative) aversion toward ambiguity as it is the weight attached to the worst possibility in the dispute set.[3]

A contract c is *ex ante efficient* if there is no other contract c' such that $V_i(c') \geqslant V_i(c)$ for $i = 1, 2$, with a strict inequality for some i.

Depending on the degree of concavity of the preference scaling functions $v_i(\cdot)$ compared to the decision-weight ambiguity aversion parameter α_i, the ambiguity may lead the parties to prefer incomplete risk sharing to possibly ambiguous contracts. This point can be illustrated with reference to the idea of liquidated damages.

3. Liquidated damages

To be effective, a contract must specify some sanction to be applied if one or other party fails to perform their obligations. In some cases, this is a relatively simple matter: failure to perform may be held to nullify the contract. In other cases, however, failure by one party to perform an obligation may cause damage to the other.

For concreteness, let us consider an example where a supplier contracts with a builder to deliver materials on a given date. However, under certain conditions, the supplier may be unable to deliver, and so may default, declaring *force majeure*. Failure to deliver on time may force the builder to source the supplies elsewhere at high cost, or to delay the project. Thus, neither nullifying the contract nor requiring (delayed) performance is an adequate remedy. The costs of failure will depend on a variety of factors. For example, rainy weather might halt construction with the result that the supplier's default causes no additional cost. In other cases, the default may occur at a crucial point in the project, creating unusually large damages.

In the absence of bounds on rationality, the parties could agree on a contract that listed all conceivable default states, and specify a payment to be made in each case. Any Pareto optimal bargaining solution in this case, derived from the state-dependent preferences of both parties, will be referred to as the first best. However, with ambiguity arising from bounded rationality, the first best may not be attainable.

One solution is for the contract to specify that the defaulting party should compensate the other party to an amount depending on the amount of their loss. In the event of a dispute over the magnitude of the loss, a court or other external arbiter will determine the payment.

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[2] We thank Roger Myerson for the suggestion that disputes can be modeled as wars of attrition.

[3] We show in Appendix A the way in which the representation in Eq. (1) arises from preferences that come from the class of expected-uncertain-utility decision-makers.

Another possibility is that of liquidated damages, in which the payment for a specific breach is fixed in the contract, without reference to the actual losses suffered by the injured party. We will look at the liquidated damages setting and give some results on when liquidated damages contracts are efficient.

We begin by assuming that $S = \Omega \times \{0, 1\}$ where the $E^d = \Omega \times \{1\}$ is interpreted as 'the situation is such that party 1 must default', and $D(s) = \{s\}$ for all $s \notin E^d$, that is, there is no ambiguity outside default. Potential disputes relate to the consequences of default, and not to the question of whether party 1 has in fact defaulted.

The action set $A = \hat{A} \times T$ is the Cartesian product of a set of actions $\hat{A}$ relevant to the performance of the contract and a set of payment actions (monetary transfers) $T = [-M, M]$. The set $\hat{A}$ is assumed to include a default action a_0. Actions $t \in T$ are interpreted as 'party 1 pays t dollars to party 2'. Actions $\hat{a} \in \hat{A} - \{a_o\}$ are prohibitively costly to party 1 in the event of default. Thus, we restrict attention to contracts that satisfy:

$$c(s) = \begin{cases} (a_c(s), t_c(s)) & \text{if } s \notin E^d \\ (a_0, t_c(s)) & \text{if } s \in E^d. \end{cases}$$

That is, the contract specifies a set of actions to be performed, and payments to be made, in the absence of default and a set of payments to be made in the presence of default. The payment $t_c(\omega, 1)$ for any $\omega \in \Omega$ (that is, a payment made in the presence of default) is referred to as a damages payment.

We further assume that, for each party $i = 1, 2$ in each state $s \in S$, the preferences over $(\hat{a}, y)$ are quasi-linear with respect to monetary transfers and so the wealth equivalent outcome in each state may be expressed in the form

$$y_i\big((\hat{a}, t), s\big) = w_i(\hat{a}, s) + (-1)^i \times t,$$

where $w_i(\hat{a}, s)$ is the monetary equivalent value to party i of the action $\hat{a}$ performed in state s.

As is standard in the literature, we assume that the preference scaling function v_i is strictly increasing ($v_i' > 0$) and strictly concave ($v_i'' < 0$), and that there is a common prior on S that we denote by π.

Notice that for any default state $s = (\omega, 1) \in E^d$, $(\omega, 0)$ is the state that would have obtained if party 1 had not defaulted. In this counter-factual state, the contract would have called for action $a_c(\omega, 0)$. Hence, for each $s = (\omega, 1) \in E^d$, $w_2(a_c(\omega, 0), s) - w_2(a_0, s)$ may be interpreted as the loss incurred by party 2 in state s, as a consequence of the default by party 1.

In the absence of ambiguity, a Pareto optimal contract c^* must satisfy the Borch condition for efficient risk-sharing, that is, the marginal rate of substitution between any pair of state-contingent payoffs must be the same for both individuals. Formally, for any pair of default states $s, s' \in E^d$.

$$\frac{\pi_s v_1'(w_1(a_0, s) - t_{c^*}(s))}{\pi_{s'} v_1'(w_1(a_0, s') - t_{c^*}(s'))} = \frac{\pi_s v_2'(w_2(a_0, s) + t_{c^*}(s))}{\pi_{s'} v_2'(w_2(a_0, s') + t_{c^*}(s'))}$$

A contract satisfying this condition will be referred to as a first-best contract. Since the contract is unambiguous in the absence of default, the first-best contract will be unambiguous if (and only if): for any pair of default states $s, s' \in E^d$ and any party $i \in \{1, 2\}$, $s' \in D(s)$ implies $w_i(a_0, s) = w_i(a_0, s')$.

Suppose, however, that the conditions relevant to the effects of default on the welfare of party 1 (the defaulting party) are ambiguous. In this case, we might instead consider the case of a contract with damages dependent on losses to party 2. Since the cardinality of E^d is finite it follows that the set

$$L_2 = \big\{\ell \in [0, M]\colon \exists \omega \in \Omega \text{ s.t. } \ell = w_2\big(a_c(\omega, 0), (\omega, 1)\big) - w_2\big(a_0, (\omega, 1)\big)\big\}$$

is also finite. Moreover, for each $\ell \in L_2$, there exists an event $E^\ell \subset S$ that obtains if and only if default occurs and the associated loss for party 2 is ℓ. That is,

$$E^\ell = \big\{\omega \in \Omega\colon\ w_2\big(a_c(\omega, 0), (\omega, 1)\big) - w_2\big(a_0, (\omega, 1)\big) = \ell\big\} \times \{1\}.$$

The members of the set of events $\{E^\ell\colon\ \ell \in L_2\} \cup \{S - E^d\}$ are mutually exclusive and exhaustive, and therefore constitute a partition of the state space. Any contract c can be amended in a way to make it a loss-dependent damages contract $\hat{c}$, by restricting it to be measurable with respect to this partition. Thus $\hat{c}$ may be specified as

$$\hat{c}(s) = \begin{cases} c(s) & \text{if } s \notin E^d \\ (a_0, t_{\hat{c}}(\ell)) & \text{if } s \in E^d, \end{cases}$$

where $t_{\hat{c}} : \mathbb{R}_+ \to [0, M]$ is a function relating the loss borne by party 2 to the associated damages payment from party 1. Note that we do not require $t_{\hat{c}}(\ell) = \ell$. That is, the damages payment from party 1 to party 2 need not be equal to the loss incurred by party 2. In typical cases, both parties will incur losses in default states, but party 2 (the 'victim' of default) will incur greater losses, and the optimal payment from party 1 will produce a more even sharing of risk.

More generally, depending on the risk-sharing properties of the contract and on the state-dependent preferences of the parties, the damages payment to party 2, $t_{\hat{c}}(\ell)$, may be less than, equal to or greater than the loss ℓ incurred by party 2.[4]

The event E^{ℓ}, however, may still be ambiguous. For example, the parties may disagree over what items should be counted as losses arising from default and how they should be valued. Thus, such contracts are likely to, and regularly do, produce disputes.

If losses are ambiguous, and dispute costs are high, parties may prefer a *liquidated damages contract* $\hat{c}$, with a specified payment $\tilde{t}$ in the case of default:

$$\tilde{c}(s) = \begin{cases} c(s) & \text{if } s \notin E^d \\ (a_0, \tilde{t}) & \text{if } s \in E^d. \end{cases}$$

That is, either the contract c, applicable in the absence of default, is implemented or the default action a_0 is undertaken and party 1 pays to party 2 the liquidated damage sum $\tilde{t}$. Since the event E^d is unambiguous, so is the liquidated damages contract.

In general, there will be gains from risk sharing across states. When the aversion to ambiguity is small across states, efficient contracts will involve risk sharing even at the ambiguous default states. However, when the aversion to ambiguity is sufficiently large, there will be no risk sharing across default states and so all efficient contracts will be liquidated damages contracts. We formally give this result in the next proposition. For ease of exposition, we presume that the possibility of dispute set $D(s)$ is the same for each default state $s \in E^d$.

Proposition 1. *Suppose that for all $s \in E^d$, $D(s) = E^d$. There is an $\bar{\alpha} < 1$ such that: if $\alpha_i > \bar{\alpha}$ for each $i \in \{1, 2\}$, then every ex ante efficient contract is a liquidated damages contract.*

Proof. Suppose a contract c is not a liquidated damages contract. Then, let $\bar{t}$ denote the maximal payment over default states under this contract, that is, $\bar{t} = \max_{s \in E^d} t_c(s)$, and let $\underline{t}$ denote the minimal payment over default states, that is, $\underline{t} = \min_{s \in E^d} t_c(s)$. Then, $\bar{t} > \underline{t}$. We will show that provided $\bar{\alpha}$ is large enough, we can increase the welfare of both parties 1 and 2 by marginally increasing $\underline{t}$ and marginally decreasing $\bar{t}$.

For this, we define:

$$a = \max_{i \in \{1,2\}} \max_{s \in E^d} v_i'\big(w_i(a_0, s) - M\big);$$
$$b = \min_{i \in \{1,2\}} \min_{s \in E^d} v_i'\big(w_i(a_0, s) + M\big);$$
$$\bar{\alpha} = \frac{a}{a+b}.$$

By assumption $v_i' > 0$ and $v_i'' < 0$, hence it follows that $a > b > 0$. We assume in what follows that: $\alpha_i > \bar{\alpha}$ for each $i \in \{1, 2\}$. The *ex ante* expected utility of 1 can be written as:

$$\begin{aligned} V_1(c) = {} & \sum_{s \notin E^d} \pi_s v_1\big(w_1\big(c(s), s\big) - t_c(s)\big) \\ & + \sum_{s \in E^d} \pi_s\big[\alpha_1 v_1\big(w_1(a_0, s) - \bar{t}\big) + (1 - \alpha_1) v_1\big(w_1(a_0, s) - \underline{t}\big)\big]. \end{aligned}$$

We consider a marginal change to $\underline{t}$ and $\bar{t}$ such that $d\underline{t} = -d\bar{t} > 0$. For such a change:

$$\begin{aligned} dV_1(c) &= \sum_{s \in E^d} \pi_s\big[\alpha_1 v_1'\big(w_1(a_0, s) - \bar{t}\big) - (1 - \alpha_1) v_1'\big(w_1(a_0, s) - \underline{t}\big)\big] d\underline{t} \\ &> \left(\sum_{s \in E^d} \pi_s\right)\big[\bar{\alpha} b - (1 - \bar{\alpha}) a\big] d\underline{t} = 0, \end{aligned}$$

with the strict inequality following from our definitions of a and b, and the properties $v_1' > 0$ and $v_1'' < 0$.

By similar reasoning for 2, we obtain $dV_2(c) > 0$. Hence, c cannot be *ex ante* efficient. □

[4] In general, risk-sharing would imply that the damages payment should be less than the loss. In the model presented here, losses are the result of *force majeure* rather than discretionary options. Hence, there is no incentive-based reason for exemplary or punitive damages. However, consideration of the state-contingent preferences of party 1 suggests instances where risk-sharing may imply a payment larger than the loss. Suppose that high-losses to party 2 occur when the good is in high demand and subject to constrained supply. Then party 1, having defaulted as a result of inability to supply on time, may be able to sell the good at a high price and therefore (involuntarily) benefit from default.

Here we see that in the case of sufficient aversion to ambiguity over default states, the optimal contract is the liquidated damages contract. The intuition in the maximally pessimistic case (that is, $\alpha_i = 1$, for both $i = 1, 2$) is as follows. Since each expects the worst in default states, we can raise the utility of 2 at all default states by raising $\underline{t}$, and simultaneously raise the utility of 1 at all default states by lowering $\bar{t}$. Since this change does not affect utility in any other state, it generates a Pareto improvement. Proposition 1 shows that intuition carries through provided the parties are sufficiently ambiguity averse.

As mentioned in the introduction, this result is similar, in important respects, to that of Mukerji and Tallon (2001) who demonstrated how probabilistic ambiguity about the idiosyncratic risk associated with financial assets may deter agents from trading such assets. In the analysis of Mukerji and Tallon, as in the decision-theoretic literature generality, ambiguity is modeled as a property of individual beliefs, namely the absence of a well-defined set of probabilities over states. By contrast, in the present paper, individual beliefs over objective states of nature are taken to be probabilistically sophisticated. As in the ordinary language use of the term, ambiguity arises in communication between the parties who may interpret the same contractual terms to refer to different objective states of nature.

Our representation of contractual disputes shows that, from a modeling viewpoint, these apparently disparate notions of ambiguity are in fact closely related. Since we have expressed linguistic ambiguity in terms of a dispute set with the preferences over contracts represented by a Gul and Pesendorfer (2014) expected uncertain utility functional, our treatment is mathematically equivalent to a party, after observing $s \in S$, having 'maximally ambiguous beliefs' about which s' in the dispute set $D(s)$ will arise as a result of the other party's interpretation of the situation. Hence, the mechanism behind the suboptimal trading result of Mukerji and Tallon is the same mechanism behind our result. This analytical connection opens the world of linguistic ambiguity to the intuitions and results of probabilistic ambiguity.

In the context of linguistic ambiguity, we have shown that sufficient (relative) aversion to ambiguity results in the parties failing to exploit all risk sharing opportunities. This occurs because the parties expect any dispute arising from a default to result in *ex post* litigation costs that burn up all surplus. By signing a 'flat' liquidated damages contract, each party commits *ex ante* to abstain from such behavior.

A natural question is whether or not a liquidated damages contract can be efficient in a standard state-space approach with contracting parties whose preferences conform to expected utility theory. The answer is that, in general, it cannot. To see this, we presume that each player will have a partition over $S \times S$ and a probability distribution over those states. Let $\rho_i(s, s')$ denote the prior probability in party i's mind that party 1 sees s and party 2 sees s'. Since we have assumed non-default is unambiguous, it follows that

$$\rho_i\big((\omega, 0), \big(\omega', 1\big)\big) = \rho_i\big((\omega, 1), \big(\omega', 0\big)\big) = 0, \quad \text{for all } \omega, \omega' \in \Omega.$$

So we shall focus on the event of default which we denote by the subset $E_d \subset S \times S$ given by $E_d = \{(s, s'):\ s, s' \in E^d\}$. Thus, the probability of a default event in the eyes of party i is $\sum_{(s,s')\in E_d} \rho_i(s, s')$. In keeping with our previous analysis, each person's wealth equivalent of an action depends only on the state each sees so when the state is (s, s') we will write $w_1(\cdot, s)$ and $w_2(\cdot, s')$. However, we allow the transfer amount t to depend on the state (s, s'). The presumption here is that some determination on (s, s') will be made *ex post* and then a transfer occurs. Each player considers each contingency (s, s') as possible when he writes the contract. Under this scenario, the *ex ante* expected utilities of parties 1 and 2 restricted to default states are respectively:

$$\sum_{(s,s')\in E_d} \rho_1\big(s, s'\big) v_1\big(w_1(a_0, s) - t\big(s, s'\big)\big) \quad \text{and} \quad \sum_{(s,s')\in E_d} \rho_2\big(s, s'\big) v_2\big(w_2\big(a_0, s'\big) + t\big(s, s'\big)\big).$$

Though typically liquidated damages will not be efficient in this full state-space approach, for a clean result we focus on a case of a common prior with a technical assumption about the richness of the state-space:

1. (*Common prior*) $\rho_1(s, s') = \rho_2(s, s') > 0$ for all $(s, s') \in E_d$.
2. (*Richness of State-space*) There are s, s' in E^d such that either $w_1(a_0, s) \neq w_1(a_0, s')$ or $w_2(a_0, s) \neq w_2(a_0, s')$.

Proposition 2. *Suppose that assumptions 1 and 2 hold. If c is ex ante efficient, then c is not a liquidated damages contract.*

Proof. Suppose c is a liquidated damages contract. By assumption 2, there are two default states s and s' where some player gets a different utility prior to any transfer. We assume, without loss of generality, that $w_1(a_0, s) \neq w_1(a_0, s')$. Consider the states (s, s) and (s', s) which are both in E_d. By assumption 1, a necessary condition for efficiency is:

$$\frac{v_1'(w_1(a_0, s) - t_c(s, s))}{v_1'(w_1(a_0, s') - t_c(s', s))} = \frac{v_2'(w_2(a_0, s) + t_c(s, s))}{v_2'(w_2(a_0, s) + t_c(s', s))}.$$

Since the contract is a liquidated damages contract, $t_c(s, s) = t_c(s', s)$, and so the right hand side of the equality must be 1. However, since $w_1(a_0, s) \neq w_1(a_0, s')$ and $v_1'' < 0$ (strict concavity), the left hand side cannot equal 1 when $t_c(s, s) = t_c(s', s)$. Since c does not satisfy the necessary condition for efficiency, c is not efficient. □

4. Concluding comments

We have provided a motivation for liquidated damages contracts in terms of linguistic ambiguity. Ambiguity can affect incentives for risk sharing, and the desirability of contracts. In particular, ambiguity may in some cases be handled effectively and efficiently by liquidated damages contracts.

The ambiguity in our model arises from multiple interpretations of the common language used to write contracts, as in Grant et al. (2012a). However, contracting is modeled in a state-space with parties whose preferences can be derived from the expected uncertain utility theory of Gul and Pesendorfer (2014), in a way that is formally equivalent to the probabilistic ambiguity approach used in the analysis of Mukerji and Tallon (2001). Hence, this representation provides a link between linguistic and probabilistic ambiguity.

The representation of ambiguity proposed here suggests new approaches to a range of issues in contract theory, and potentially broader applications in agency theory. The standard principal–agent problem is one where contracting is limited to some observable unambiguous characteristics like output, rather than a full set of characteristics, including effort levels, which may be ambiguous. The framework developed here suggests the possibility of an endogenous choice between contracts over different characteristics, where the choice of the contractual variables depends on the level of ambiguity and potential gains from risk sharing. While this application would involve some new technical complications regarding the appropriate treatment of tests, the benefit would be the development of a theory of contracting in which the terms of the contract, over which the parties actually bargain, plays the central role.

Appendix A. Relationship to expected uncertain utility theory

To see how the preferences over contracts of each party generated by the function given in Eq. (1) may be viewed as having come from an expected-uncertain-utility decision-maker, recall that an expected-uncertain-utility decision-maker is characterized by a *prior* and an *interval utility*. The prior comprises both an algebra of (measurable) events of the state-space as well as a probability measure defined on that algebra. For the prior of each party, we take the algebra $\mathcal{E}$ of (measurable) events to be an algebra of subsets of the *product* state-space $S \times S$ generated from the partition

$$\big\{\{s\} \times D(s)\colon\ s \in S\big\} \cup \big\{\{s\} \times S \backslash D(s)\colon\ s \in S\big\}.$$

Since each party presumes the other will see something within her or his own dispute set, the measure μ defined on $\mathcal{E}$ satisfies the following: for all s in S,

$$\mu\big(\{s\} \times D(s)\big) = \pi_s \quad \text{and} \quad \mu\big(\{s\} \times S \backslash D(s)\big) = 0.$$

For each party $i = 1, 2$, we assume her interval utility function $u_i(x, x')$, where $x \leqslant x'$, is additively separable and given by $u_i(x, x') = \alpha_i v_i(x) + (1 - \alpha_i) v_i(x')$. Furthermore, we associate with each contract $c : S \to A$ the 'act' $f_i^c : S \times S \to \mathbb{R}$, given by

$$f_i^c\big(s, s'\big) = \begin{cases} y_i(c(s), s) & \text{if } (*) \text{ holds} \\ y_i(c(s'), s) & \text{otherwise,} \end{cases}$$

where $(*)$ is said to hold if either

$$\textit{(i)} \quad y_{(3-i)}\big(c(s), s'\big) > y_{(3-i)}\big(c\big(s'\big), s'\big),$$

or

$$\textit{(ii)} \quad y_{(3-i)}\big(c(s), s'\big) = y_{(3-i)}\big(c\big(s'\big), s'\big) \quad \text{and} \quad y_i\big(c(s), s\big) > y_i\big(c\big(s'\big), s\big).$$

Following expression (2) of Gul and Pesendorfer (2014, p. 1), we define the utility of an act $f : S \times S \to \mathbb{R}$ for party i to be given by

$$U_i(f) = \int u\big([f]_1, [f]_2\big)\, d\mu,$$

where $[f]_1$ (respectively, $[f]_2$) is the supremum (respectively, infimum) of the set of acts measurable with respect to the algebra $\mathcal{E}$ that are dominated by (respectively, dominate) f.

Using the expression for $V_i(c)$ given in Eq. (1), by straightforward (albeit tedious) calculation we obtain: $V_i(c) = U_i(f_i^c)$, for all contracts c in C.

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